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XIV.

ON THE SOLAR MOTION AND THE STELLAR DISTANCES.

By Truman Henry Safford.

(*Third Paper.*)

Presented, Oct. 12, 1875.

In previous papers, I have given investigations of this subject, based upon proper motions determined by others. In the present, I give the formulæ and tables necessary for the farther prosecution of the subject, in a form slightly different from the ordinary one. In view of previous investigations by Mädler and Kovalski, it will be necessary to inquire very minutely into the proper motions whose annual amount is about 0.″1. The more swiftly moving stars (as we see them) have been investigated by Argelander and other astronomers, and will be carefully tested: some of them, it is possible, will show a motion not directly proportional to the time, not only because the great circle in which the star appears to move does not meet the successive meridians at angles varying in the manner in which this hypothesis would require, but also because we are nearing or receding from them, relatively considered. Thus the terms depending on squares and products of proper motion, which Mr. G. W. Hill has, I believe, first introduced, which are the result of the varied relation between the star's apparent motion and any set of *fixed* planes, will not perhaps be the only terms of this kind; so that in such investigations I prefer to omit them and determine by observation the total amount of these terms, following in this matter the example set by Bessel.

But the proper motions between 0″.1 and 0″.2 are susceptible in many cases (not yet studied) of accurate determination from observations now extant; and in many other cases a few additional observations will be sufficient, so that the work immediately needed is to select from the whole mass such stars of this class as are adapted to either method of treatment.

The same is true in a greater or less degree of the stars whose annual proper motions are less than 0″.1. If these are Bradley's stars, the new reduction by Professor Auwers, and redeterminations at Pulcova and Greenwich, will be sufficient for the present; but, if not, so wide a field for minute criticism is thus opened, that I suspect the only cases at present worth testing will be those in which special accuracy is to be expected; as in the vicinity of the north pole, where the early observations of W. Struve afford the best possible means of comparison, in addition to the standard places of Groombridge.

In what follows, I give first the precession-constants and formulæ; and next an auxiliary table for the computation of the relations between the star's proper motion and the solar motion supposed directed to the point whose AR. is 259°50′.8 and Decl. + 32°29′.1.

The computation of the most probable proper motion involves: first, the reduction of all observations with proper systematic corrections to a fixed epoch by precession, next the assignment of weights and establishment of conditional equations, and lastly their solution; but when the proper motion is large, or the star near the pole, either the geometrical formulæ must be used, or a preliminary proper motion employed to compute the terms of secular variation depending on it. In volume IV. Part I. of the Annals of Harvard College Observatory (also included in Volume VIII. of the Memoirs of this Academy, New Series), I have given some examples of a still more rigid treatment of such cases.

PRECESSION-CONSTANTS AND FORMULÆ.

Fundamenta Astronomiæ, p. 297 (Bessel, I.).

For $1750 + t$

$$m = 45''.99592 + t\,0''.0003086450$$
$$n = 20\,.05039 - t\,0\,.0000970204$$

Annales de l'Observatoire Impériale (Mémoires, II. 209).

For $1850 + t_0$

$$m\ (\mu) = 46''.05912 + 0''.00028372\ t_0$$
$$n\ (\nu) = 20\,.05197 - 0\,.00008663\ t_0$$

Tabulæ Regiomontanæ (Bessel II.).

For $1750 + t$

$$m = 46''.02824 + t\,0''.0003086450$$
$$n = 20\,.06442 - t\,0\,.0000970204$$

Peters, Numerus constans Nutationis, p. (195).

For 1800 + t

$$m = 46.''0623 + 0''.0002849\,t$$
$$n = 20\;.0607 - 0\;.0000863\,t$$

	Bessel II.				Struve — Peters.		
	m	$\frac{m}{15}$	n	Log. n	m	n	Log. n
1750	46''.0282	$3^{s}.06855$	20''.0644	1.302427	46''.0481	20''.0650	1.302439
60	.0313	.06876	.0634	406	.0509	.0642	421
70	.0344	.06896	.0625	385	.0538	.0633	402
80	.0375	.06917	.0615	364	.0566	.0624	383
90	.0406	.06937	.0605	343	.0595	.0616	365
1800	.0437	.06958	.0596	322	.0623	.0607	346
10	.0468	.06978	.0586	301	.0651	.0598	327
20	.0498	.06999	.0576	280	.0680	.0590	309
30	.0529	.07020	.0567	259	.0708	.0581	290
40	.0560	.07040	.0557	238	.0737	.0572	271
50	.0591	.07061	.0547	217	.0765	.0564	253
60	.0622	.07081	.0537	196	.0794	.0555	234
70	.0653	.07102	.0528	175	.0822	.0547	215
80	.0684	.07122	.0518	154	.0851	.0538	197
90	.0715	.07143	.0508	133	.0879	.0529	178
1900	46.0745	3.07164	20.0499	1.302112	46 .0908	20 .0521	1.302159

The formulæ for secular variation are, as given by Menten, —

$$100\,\frac{d^2\alpha}{dt^2} = A + B \tan\delta + C \tan\delta^2$$

$$100\,\frac{d^2\delta}{dt^2} = A' + B' \tan\delta$$

in which, employing Bessel's constants for 1860, —

$$A = 0^{s}.00206 + 0^{s}.00650 \sin 2\,\alpha = 0^{s}.00206 + \tfrac{1}{2}\,C$$
$$B = [8.4750] \cos\alpha + [6.811\,n] \sin\alpha$$
$$C = [8.1139] \sin 2\,\alpha$$

$$A' = -\,0.''0097 \cos\alpha - 0.''4479 \sin\alpha$$
$$B' = [9.2900\,n] \sin\alpha^2.$$

Menten has tabulated the values of A and A', and the logarithms of the other quantities for every minute of right ascension. These expressions do not include the effect either of precession in changing the apparent amount of proper motion, or of proper motion in changing the precession; in other words, they are geometrical secular variations only. The secular variation of the whole motion, omitting terms of the order of the squares and products of the proper motions, is obtained by adding to Menten's formula the terms, —

$$0.01944 \left[\mu \tan \delta \cos \alpha + \frac{\mu'}{15} \sec \delta^2 \sin \alpha\right]$$

$$- 0.01944.\ 15\ \mu \sin \alpha$$

in right ascension and declination respectively; where μ and μ' are the annual proper motions, μ expressed in time, μ' in arc of a great circle.

Argelander in his Åbo catalogue gives secular variations of precession only, and includes one half of these terms. Thus for σ Draconis, the geometrical secular variations for 1830, computed by Menten's formulæ (without allowing for the trifling secular change in the table), are, —

$$- 0^{s}.03622 \text{ and } - 0''.0299.$$

The proper-motion terms in the secular variation of the whole motion, according to the previous formulæ, are, —

$$+ 0^{s}.01921 + 0''.0254.$$

Hence the secular variation of the total motion is, for 1830, —

$$- 0^{s}.01701 \text{ and } - 0''.0045,$$

not including squares and products of proper motion.

The Åbo Catalogue has, —

$$- 0^{s}.0265 \text{ and } - 0''.017;$$

that is, one half the proper-motion terms in the secular variation of the whole motion, or, in other words, the whole effect of proper motion in changing the precession, is included.

Sometimes it is necessary to employ the geometrical formulæ: they are these, according to Carrington, using Bessel's constants, for the reduction from 1750 + t to 1750 + t'.

$$z + \lambda = 23''.0144\ (t' - t\ ) - 0''.0115\ (t' + t) - 0''.000056\ t'^2 - 0''.000210\ t^2$$

$$z' - \lambda' = 23''.0144\ (t' - t) + 0''.0115\ (t' + t) + 0''.000210\ t'^2 + 0''.000056\ t^2$$

$$\theta = 20''.0652\ (t' - t) - 0''.000048\ t'^2 + 0''.000048\ t^2$$

$$a = \alpha + z + \lambda$$

$$\begin{aligned} \cos \delta' \sin a' &= \cos \delta \sin a \\ \cos \delta' \cos a' &= \cos \delta \cos a \cos \theta - \sin \delta \sin \theta \\ \sin \delta' \quad &= \cos \delta \cos a \sin \theta + \sin \delta \cos \theta \end{aligned}$$

$$\alpha' = a' + z' - \lambda'$$

Using Struve's constants, the values become: —

$$\begin{aligned} z + \lambda &= 23''.0311\ (t' - t) - 0''.0001922\ t^2 - 0''.0000497\ t'^2 \\ z' - \lambda' &= 23\ .0311\ (t' - t) + 0\ .0000497\ t^2 + 0\ .0001922\ t'^2 \\ \theta \quad &= 20\ .0611\ (t' - t) + 0\ .0000432\ t^2 - 0\ .0000432\ t'^2 \end{aligned}$$

According to Bessel II., the formulæ for the computation of these quantities are, —

$$\begin{aligned} \lambda &= \qquad 0''.17926\ t - 0''.0002660394\ t^2 \\ \omega &= 23^\circ 28' 18''.0 + 0''.00000984233\ t^2 \\ \psi &= 50''.37572\ t - 0\ .0001217945\ t^2; \end{aligned}$$

λ', ω', and ψ', are the same functions of t'.

$$\tfrac{1}{2}\ (z' - z) = 0''.1011804\ (t' + t) + 0''.0000002446\ (t' + t)^2$$

$$\tan \tfrac{1}{2}\ (z' + z) = \frac{\cos \frac{1}{2}\ (\omega' + \omega)}{\cos \frac{1}{2}\ (\omega' - \omega)} \tan \tfrac{1}{2}\ (\psi' - \psi)$$

$$\tan \tfrac{1}{2}\ \theta = \frac{\sin \frac{1}{2}\ (z' + z)}{\cos \frac{1}{2}\ (z' - z)} \tan \tfrac{1}{2}\ (\omega' + \omega)$$

The time is counted from 1750.

According to Struve and Peters, the time is counted from 1800,

$$\begin{aligned} \text{and } \lambda &= \qquad 0''.15119\ t - 0''.00024186\ t^2 \\ \omega &= 23^\circ 27' 54''.45 \qquad + 0\ .00000735\ t^2 \\ \psi &= \qquad 50\ .3798\ t \ - 0\ .0001084\ t^2 \\ \tfrac{1}{2}\ (z' - z) & \qquad\qquad = 0\ .075573\ (t' + t) \\ & \qquad\qquad + 0\ .0000001626\ (t' + t)^2 \end{aligned}$$

The last formula is quoted from Carrington.

LeVerrier counts his time from 1850, and finds (I employ here Bessel's notation), —

$$\begin{aligned}
\tfrac{1}{2}\,(\omega' + \omega) &= 23^\circ 27' 32''0 + 0''.00000360\,(t^2 + t'^2) \\
\tfrac{1}{2}\,(\psi' - \psi) &= 25''.18516\,(t' - t) - 0''.00005440\,(t'^2 - t^2) \\
\tfrac{1}{2}\,(z' - z) &= 0\,.07396\,(t' + t) + 0\,.00000016\,(t' + t)^2 \\
\lambda &= 0\,.14790\,t - 0\,.00024167\,t^2
\end{aligned}$$

TABLE FOR *M* AND *m*.

[*See page* 53 *of this Volume.*]

a	*M*	log. sin *m*	log. cos *m*
0°	105°.45	9.7460	9.9193
2	108.27	9.7525	9.9163
4	110.99	9.7598	9.9127
6	113.58	9.7679	9.9086
8	116.05	9.7766	9.9040
10	118 40	9.7857	9.8987
12	120.62	9.7952	9.8928
14	122.71	9.8050	9.8864
16	124.68	9.8150	9.8793
18	126.53	9.8250	9.8715
20	128.26	9.8350	9.8630
22	129.88	9.8450	9.8539
24	131.39	9.8549	9.8440
26	132.80	9.8645	9.8333
28	134.12	9.8740	9.8218
30	135.35	9.8832	9.8095
32	136.50	9.8922	9.7962
34	137.56	9.9009	9.7820
36	138.55	9.9092	9.7668
38	139.47	9.9172	9.7505
40	140.32	9.9249	9.7329
42	141.11	9.9322	9.7141
44	141.84	9.9392	9.6939
46	142.52	9.9458	9.6721
48	143.14	9.9520	9.6486
50	143.71	9.9579	9.6232
52	144.24	9.9633	9.5958
54	144.72	9.9684	9.5658
56	145.15	9.9731	9.5331
58	145.55	9.9774	9.4971

TABLE FOR M AND m (*continued*).

a	M	log. sin m	log. cos m	m
60	145°.90	9.9814	9.4573	73°.34
62	146.22	9.9850	9.4129	75.00
64	146.50	9.9881	9.3628	76.67
66	146.74	9.9909	9.3056	78.34
68	146.95	9.9934	9.2391	80.01
70	147.14	9.9954	9.1598	81.69
72	147.27	9.9971	9.0622	83.37
74	147.38	9.9984	8.9354	85.06
76	147.46	9.9993	8.7546	86.74
78	147.50	9.9998	8.4382	88.43
80	147.51	0.0000	7.3036 n	90.12
82	147.50	9.9998	8.4976 n	91.80
84	147.45	9.9992	8.7842 n	93.49
86	147.37	9.9982	8.9551 n	95.17
88	147.25	9.9969	9.0770 n	96.86
90	147.10	9.9952	9.1716 n	98.54
92	146.92	9.9931	9.2488 n	100.21
94	146.71	9.9906	9.3139 n	101.89
96	146.46	9.9877	9.3700 n	103.56
98	146.18	9.9845	9.4193 n	105.22
100	145.86	9.9809	9.4630 n	
102	145.50	9.9769	9.5022 n	
104	145.10	9.9725	9.5377 n	
106	144.66	9.9677	9.5701 n	
108	144.17	9.9626	9.5996 n	
110	143.64	9.9571	9.0269 n	
112	143.06	9.9512	9.6520 n	
114	142.43	9.9449	9.6752 n	
116	141.75	9.9383	9.6967 n	
118	141.01	9.9313	9.7168 n	
120	140.21	9.9239	9.7354 n	
122	139.35	9.9162	9.7528 n	
124	138.42	9.9081	9.7690 n	
126	137.42	9.8997	9.7840 n	
128	136.35	9.8910	9.7981 n	
130	135.19	9.8820	9.8112 n	
132	133.95	9.8727	9.8234 n	
134	132.62	9.8632	9.8348 n	
136	131.19	9.8535	9.8454 n	
138	129.66	9.8436	9.8552 n	
140	128.03	9.8337	9.8642 n	
142	126.28	9.8236	9.8726 n	
144	124.42	9.8136	9.8802 n	
146	122.43	9.8037	9.8873 n	
148	120.32	9.7939	9.8937 n	

TABLE FOR M AND m (*continued*).

a	M	log. sin m	log. cos m
150	118°.09	9.7844	9.8995 n
152	115.72	9.7754	9.9046 n
154	113.23	9.7668	9.9092 n
156	110.62	9.7588	9.9133 n
158	107.89	9.7516	9.9167 n
160	105.06	9.7452	9.9196 n
162	102.13	9.7398	9.9220 n
164	99.12	9.7356	9.9238 n
166	96.03	9.7324	9.9251 n
168	92.92	9.7306	9.9259 n
170	89.79	9.7300	9.9261 n
172	86.65	9.7308	9.9258 n
174	83.53	9.7328	9.9250 n
176	80.47	9.7361	9.9236 n
178	77.47	9.7405	9.9217 n
180	74.55	9.7460	9.9193 n

$$\cos \chi = \sin m \cos (\delta - M)$$
$$\sin \chi \cos \psi' = \sin m \sin (\delta - M)$$
$$\sin \chi \sin \psi' = \cos m.$$

If α exceeds 180°, cos m changes sign, and the supplements of m and M are given in the table with argument $\alpha - 180°$.